MEHRDAD NAHALPARVARI
FIXED SWITCHING FREQUENCY DIRECT MODEL PREDICTIVE CONTROL FOR GRID-CONNECTED CONVERTERS

Master of Science thesis

Examiner: Assistant Professor Petros Karamanakos
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ABSTRACT

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Model predictive control (MPC) has recently been gaining ground as a suitable control method for power electronic converters. It is formulated as an optimization problem in the time domain subject to certain constraints. Moreover, it is able to handle multiple-input multiple-output (MIMO) switched (non)linear systems and implement limitations in the form of hard and/or soft constraints.

The important challenges in controlling grid-connected converters involve ensuring that the output harmonic spectra comply with specific grid codes and that fast responses are achieved during changes in power references. In grid-connected applications, most often the produced harmonic spectra need to be well-defined to meet the grid codes. A fixed switching frequency and a symmetrical switching pattern ensure discrete harmonic spectra, making the compliance with the grid codes easier. In addition, fast responses during transients can be achieved by eliminating the modulator, i.e., direct control.

This thesis presents a direct MPC algorithm for a three-phase two-level grid-connected voltage source converter (VSC) with an LCL filter that can operate the converter at a fixed switching frequency despite the absence of a modulator. The performance of the proposed method is compared to open-loop carrier-based pulse width modulation (CB-PWM) and the IEEE519 grid code. Several refinements to the algorithm are presented which improve the performance of the system. Moreover, the algorithm is extended to emulate the 120° discontinuous PWM switching pattern. In steady-state operation, the method achieves similar total harmonic distortion (THD) levels to CB-PWM, and during transients faster responses are obtained due to the elimination of the modulation stage.
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I dedicate this thesis to my dear parents whom without their love, support and encouragement, I would not be who I am today.

Tampere, 12.09.2018

Mehrdad Nahalparvari

بی‌درود

“Remember the flight. The bird is mortal.”
- Forough Farrokhzad
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<th>Description</th>
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<tbody>
<tr>
<td>ac</td>
<td>alternating current</td>
</tr>
<tr>
<td>CB-PWM</td>
<td>carrier-based pulse width modulation</td>
</tr>
<tr>
<td>dc</td>
<td>direct current</td>
</tr>
<tr>
<td>DFT</td>
<td>discrete Fourier transform</td>
</tr>
<tr>
<td>DPWM</td>
<td>discontinuous pulse width modulation</td>
</tr>
<tr>
<td>FCS-MPC</td>
<td>finite control set model predictive control</td>
</tr>
<tr>
<td>LV</td>
<td>low voltage</td>
</tr>
<tr>
<td>MPC</td>
<td>model predictive control</td>
</tr>
<tr>
<td>MIMO</td>
<td>multiple-input multiple-output</td>
</tr>
<tr>
<td>ODE</td>
<td>ordinary differential equation</td>
</tr>
<tr>
<td>p.u.</td>
<td>per unit</td>
</tr>
<tr>
<td>PWM</td>
<td>pulse width modulation</td>
</tr>
<tr>
<td>QP</td>
<td>quadratic program</td>
</tr>
<tr>
<td>rms</td>
<td>root mean square</td>
</tr>
<tr>
<td>SISO</td>
<td>single-input single-output</td>
</tr>
<tr>
<td>THD</td>
<td>total harmonic distortion</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>zero vector</td>
</tr>
<tr>
<td>$A$</td>
<td>discrete-time state-space system matrix</td>
</tr>
<tr>
<td>$B$</td>
<td>discrete-time state-space input matrix</td>
</tr>
<tr>
<td>$C$</td>
<td>discrete-time/continuous-time state-space output matrix</td>
</tr>
<tr>
<td>$D$</td>
<td>continuous-time state-space system matrix</td>
</tr>
<tr>
<td>$e$</td>
<td>matrix exponential</td>
</tr>
<tr>
<td>$E$</td>
<td>continuous-time state-space input matrix</td>
</tr>
<tr>
<td>$e_{\text{rms}}$</td>
<td>squared root mean square error</td>
</tr>
<tr>
<td>$f_1$</td>
<td>fundamental frequency</td>
</tr>
<tr>
<td>$f_c$</td>
<td>carrier frequency</td>
</tr>
<tr>
<td>$f_{\text{res}}$</td>
<td>resonance frequency of the filter</td>
</tr>
<tr>
<td>$f_{\text{sw}}$</td>
<td>average switching frequency</td>
</tr>
<tr>
<td>$i_g$</td>
<td>grid current in $\alpha\beta$</td>
</tr>
<tr>
<td>$i_{g,abc}$</td>
<td>grid current in $abc$</td>
</tr>
<tr>
<td>$i_{\text{conv}}$</td>
<td>inverter current in $\alpha\beta$</td>
</tr>
<tr>
<td>$i_{\text{conv,abc}}$</td>
<td>inverter current in $abc$</td>
</tr>
<tr>
<td>$I$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$J$</td>
<td>objective function</td>
</tr>
<tr>
<td>$k$</td>
<td>discrete-time time-step</td>
</tr>
</tbody>
</table>
\( k_{sc} \) short-circuit ratio
\( k_{XR} \) grid impedance ratio
\( \overline{K} \) reduced Clarke transformation matrix
\( \overline{K}^{-1} \) reduced inverse Clarke transformation matrix
\( \overline{K}(\varphi) \) reduced Park transformation matrix
\( \overline{K}^{-1}(\varphi) \) reduced inverse Park transformation matrix
\( L_g \) grid inductance
\( m \) modulation index
\( \mathbf{m} \) vector of constant slopes in \( \alpha\beta \)
\( N_p \) prediction horizon length
\( Q \) weighting matrix of the objective function
\( Q' \) weighting matrix of the discrete time steps
\( R_c \) parasitic resistance of the filter capacitance
\( R_g \) grid resistance
\( R_{gr} \) equivalent grid-side resistance of the system
\( R_{ic} \) converter-side filter resistance
\( R_{ig} \) grid-side filter resistance
\( \mathbf{R}(\varphi) \) rotation matrix
\( \mathbf{R}^{-1}(\varphi) \) inverse rotation matrix
\( S_{nom} \) rated power of the converter
\( S_{sc} \) short-circuit power
\( t \) time to switch
\( \mathbf{t} \) vector of times to switch
\( T_c \) carrier period
\( T_1 \) fundamental period
\( T_s \) sampling period
\( \hat{u}^* \) peak value of the scaled voltage reference
\( \mathbf{u}_{abc} \) input vector, vector of three-phase switch positions
\( \mathbf{u}^* \) scaled three-phase voltage reference
\( \mathbf{U} \) sequence of control actions/inputs
\( \mathbf{v} \) converter output voltage in \( \alpha\beta \)
\( v_0 \) common-mode voltage
\( \hat{v} \) amplitude of the voltage reference
\( \mathbf{v}^* \) three-phase voltage reference signal
\( \mathbf{v}_c \) capacitor voltage in \( \alpha\beta \)
\( \mathbf{v}_{c,abc} \) capacitor voltage in \( abc \)
\( \mathbf{v}_g \) grid voltage in \( \alpha\beta \)
\( \mathbf{v}_{g,abc} \) grid voltage in \( abc \)
\( V_{dc} \) dc-link voltage
\( V_g \) line-to-line rms grid voltage
\( \mathbf{x} \) state vector
\( X_c \) reactance of the filter capacitance
\( X_g \) grid reactance
\( X_{gr} \) equivalent grid-side reactance of the system
\( X_{lc} \) converter-side filter reactance
\( X_{lg} \) grid-side filter reactance
\( \mathbf{y} \) output vector
\( Z_g \) grid impedance
\( \omega_{fr} \) angular speed of the reference frame
\( \omega_g \) angular frequency of the grid
\( \varphi \) angular position of a reference frame
1. INTRODUCTION

Conventional control of three-phase grid-connected converters involves using linear proportional integral (PI) controllers. Since PI controllers are single-input single-output (SISO), cascaded control loops need to be utilized in order to control the system; an inner current control loop which controls the active and reactive power by controlling the grid current, and an outer voltage control loop which controls the dc-link voltage by manipulating the real power and provides the reference values for the inner loop [1]. Controlling the grid current is achieved by manipulating the three-phase converter voltage. As the cascaded loops may adversely interfere with each other, extra effort has to be made in tuning the controllers so as to prevent instability [2]. On the other hand, model predictive control (MPC) is able to handle multiple-input multiple-output (MIMO) switched (non)linear systems. Thus, the usage of cascaded loops is circumvented. Avoiding cascaded loops is beneficial in a sense that faster dynamic responses are achieved during transients and the tuning stage is more straightforward [3].

Finite control set MPC (FCS-MPC) is a well-known MPC strategy which exploits the discrete switching nature (i.e., the finite number of switching states) of power converters. The modulation stage is eliminated in FCS-MPC. Thus, fast transient responses are achieved at the expense of a variable switching frequency and a non-discrete harmonic spectrum [4]. Therefore, FCS-MPC is not suitable for grid-connected converters as meeting the grid codes for harmonic emissions is challenging when having a spread harmonic spectrum. Furthermore, FCS-MPC requires a high sampling frequency in order to obtain desirable performance in terms of reference tracking and total harmonic distortion (THD). This may not be practical, especially when having long prediction horizons, because of the excessive computational burden.

Some strategies have been proposed in order to address the drawback of variable switching frequency and the non-discrete harmonic spectrum in FCS-MPC. In [5], a harmonic spectrum shaping method is proposed which utilizes a narrow-band stop filter in the objective function. Although with this method the range of the harmonics is limited, the switching frequency is not constant. Other attempts include
deadbeat control concepts which feed reference signals to a modulator [6,7]. However, employing a modulation stage deteriorates the fast dynamic response during transients which is inherent to non-modulator based schemes.

In conventional FCS-MPC, switching is possible only at the sampling instants. This could result in high ripple in the controlled variables, especially at rather low sampling frequencies. To overcome this, direct MPC strategies are proposed which allow the switching transitions to take place at any time instant within the sampling interval [8,11]. Consequently, the controller is allowed to choose the best possible switching instant and the ripple in the controlled variables is dealt with. This is made possible by introducing the switching instants as optimization variables. In [8], a variable switching point predictive current control (VSP²CC) for the quasi-Z-source inverter (qZSI) is proposed which computes the optimal pair of switching instants and switching sequence at one stage. However, the generated switching pattern is not symmetrical, implying a non-discrete harmonic spectrum. A modulated MPC strategy is presented in [9] which operates a seven-level cascaded H-bridge back-to-back converter at a fixed switching frequency. Nonetheless, the optimal switch positions and switching times are computed at two separate stages, leading to suboptimal results. In addition, the produced harmonic spectrum is not discrete, making the algorithm unsuitable to adopt for grid-side converter applications. In [10], an FSC-MPC scheme is proposed which ensures a fixed switching frequency. The performance of the scheme is investigated experimentally on a five-level flying capacitor converter. Nevertheless, the produced harmonic spectrum with this scheme is also non-discrete. An improved MPC algorithm with active power ripple minimization is proposed in [11] which suffers from the same drawbacks as in [8]. In [12,13], direct power control algorithms were proposed which yield a fixed switching frequency by considering pre-computed switching sequences. Nonetheless, [12,13] are limited to problems which are unconstrained with respect to time.

In [14] and [15], a direct MPC algorithm for a variable speed drive system was proposed. By constraining each phase leg to switch once per sampling interval similar to pulse width modulation (PWM), a fixed switching frequency as well as a discrete harmonic spectrum was ensured, resolving the issues of [8,11]. This thesis adopts the proposed algorithm for controlling a three-phase two-level grid-connected converter with an LCL filter. The controlled variables are the converter and the grid currents and the capacitor voltage. In each sampling interval, an optimization problem is solved, the solution of which is the optimal switching sequence to be applied at the optimal switching instants to the converter. Several refinements are presented which contribute to the improved performance of the system as quantified by the grid current THD. Moreover, the algorithm is extended to emulate the switching
pattern of 120° discontinuous PWM. The discontinuous scheme allows lowering the switching frequency and thus, the switching losses while meeting the grid codes, at the expense of a slightly higher THD for a given switching frequency. Finally, the compliance of the produced grid current harmonic spectra with the IEEE519 grid code is investigated [16].

The thesis consists of five chapters: Chapter 2 presents the theoretical background required for this thesis. Chapter 3 describes the formulation of the direct MPC algorithm and its corresponding optimization problem in detail. Chapter 4 evaluates the performance of the proposed direct MPC algorithm and compares it to the IEEE519 grid code and open-loop CB-PWM. Finally, Chapter 5 draws conclusions based on the obtained results and presents suggestions for possible future research.
2. THEORETICAL BACKGROUND

This chapter presents the theoretical background required for this thesis. Fundamental concepts behind converter control and MPC are discussed in detail. A brief introduction on conventional control strategies and carrier-based pulse width modulation (CB-PWM) is also presented.

Section 2.1 presents the state-space model of the power electronic system investigated in this thesis. Section 2.2 briefly discusses conventional control strategies used in controlling power electronic converters. The concept of CB-PWM and the harmonic analysis associated with it are presented in Section 2.3. Section 2.4 covers the key concepts of mathematical optimization. Finally, Section 2.5 introduces the concept of MPC.

2.1 Case Study

This section presents the state-space model of a power electronic system. In power electronics, the system model can be assumed as linear when voltages or currents are chosen as state and output variables. The dynamic model of the system to be controlled can be defined by the ordinary differential equations (ODEs) of the state, input and output variables. The dynamic evolution of the system in continuous-time domain can be described by the state-space representation

\[
\frac{dx(t)}{dt} = Dx(t) + E \tilde{K} u_{abc}(t) \\
y(t) = Cx(t),
\]

where \(x \in \mathbb{R}^{n_x}\) and \(y \in \mathbb{R}^{n_y}\) are the state and output vectors, respectively, and \(n_x, n_y \in \mathbb{N}^+\). Furthermore, the input vector \(u_{abc} = [u_a \ u_b \ u_c]^T \subset \mathbb{Z}^3\) is the three-phase switch position. Moreover, \(D\), \(E\) and \(C\) are the state, input and output matrices, respectively, which characterize the system. The reduced Clarke transformation matrix \(\tilde{K}\) maps the variables in \(abc\)-plane into the stationary orthogonal \(\alpha\beta\) reference frame. The stationary and rotating orthogonal reference frames are explained in Appendix A.
2.1. Case Study

In this thesis, all variables in the abc-plane are denoted by their corresponding subscript, whereas the subscript is dropped for those in the \( \alpha \beta \)-plane. Moreover, the modeling and the simulation has been done in per unit. The per unit system and the base values are introduced in Appendix B.

Using exact Euler discretization, the discrete-time state-space model of the system can be derived as

\[
x(k + 1) = Ax(k) + B \tilde{K} u_{abc}(k) \\
y(k) = C x(k),
\]

with \( A = e^{DT_s} \) and \( B = -D^{-1}(I - A)E \). Moreover, \( I \) is the identity matrix, \( e \) the matrix exponential, \( T_s \) the sampling interval and \( k \in \mathbb{N} \).

The grid-tied converter system is shown in Figure 2.1. It consists of six main parts: the power supply, the dc-link, the converter, an optional step-down transformer, the filter and the grid. The power supply is denoted by \( V_{dc} \). The dc-link capacitor serves as energy storage and maintains an almost constant voltage on the dc side. The converter converts the dc input to a three-phase ac output with variable amplitude and frequency or vice versa. A step-down transformer is sometimes used to reduce the voltage fed to the converter by the grid. A filter is responsible to attenuate the harmonics produced by the power electronic converter.

This thesis studies only the control stage of the grid-connected converter. The dc-link voltage is assumed to be constant and the transformer is neglected since the investigated case study deals with low voltages.
2.1.1 Three-Phase Two-Level Grid-Connected Converter With LCL Filter

As mentioned before, the converter converts dc energy into ac or vice versa. From various converter topologies, the two-level topology is chosen to be studied in this thesis, due to its rather low switching losses in low voltage applications. The conventional three-phase two-level converter topology depicted in Figure 2.2 consists of three legs, each connected to one of the three phases, with two switches on each leg. Each switch has an anti-parallel diode to allow bidirectional current flow and provide protection.

The output potential of each phase of the two-level converter can take the two discrete voltage levels \(-\frac{V_d}{2}\) and \(\frac{V_d}{2}\), depending on the switch position in the respective phase. Consequently, the converter output voltage \(v_{\text{conv}}\) is given by

\[
v_{\text{conv}} = \frac{V_d}{2} K u_{abc}, \tag{2.3}\]

where \(u_{abc} = [u_a u_b u_c]^T \in \{-1, 1\}^3\) is the three-phase switch position.

The converter can produce a total of \(2^3 = 8\) different switching combinations which correspond to eight voltage vectors; six active and two zero voltage vectors. The zero vectors correspond to the states where either all three upper or all three lower switches are on simultaneously. The eight possible voltage vectors are illustrated in Figure 2.3.

In grid-connected applications, it is common practice to utilize LCL filters due to the high level of harmonic attenuation they provide [17]. Figure 2.4 shows the equivalent circuit of the grid-connected converter equipped with an LCL filter.
2.1. Case Study

\[ u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \end{bmatrix}^T \in \{-1, 1\}^7 \]

\[ y = \begin{bmatrix} i_{conv} & i_g & v_c \end{bmatrix}^T \in \mathbb{R}^6. \]

Figure 2.3 Two-level converter voltage vectors in stationary reference frame.

**LCL Filter Equipped Converter Model**

The converter and the grid currents along with the capacitor and the grid voltages have been chosen as the state variables, i.e.,

\[ x = \begin{bmatrix} i_{conv} & i_g & v_c & v_g \end{bmatrix}^T \in \mathbb{R}^8. \]

The grid voltage is chosen as a state variable and not a disturbance since its evolution also needs to be predicted in MPC algorithm explained in Chapter 3.

The input vector comprises the set of three-phase switch positions in the abc-plane, and the converter current, the grid current and the capacitor voltage in the αβ-plane are considered as the output, i.e.,

\[ u_{abc} = \begin{bmatrix} u_a & u_b & u_c \end{bmatrix}^T \in \{-1, 1\}^3 \]

\[ y = \begin{bmatrix} i_{conv} & i_g & v_c \end{bmatrix}^T \in \mathbb{R}^6. \]
2.1. Case Study

![Figure 2.4 Equivalent circuit of the LCL filter equipped grid-connected converter.](image)

The discrete-time state $D$, input $E$ and output $C$ matrices can be defined as

$$
D = \begin{bmatrix}
-\frac{R_{lc}+R_c}{X_{lc}} I_2 & \frac{R_c}{X_{lc}} I_2 & \frac{1}{X_{lc}} I_2 & 0_{2\times2} \\
\frac{R_c}{X_{gr}} I_2 & -\frac{R_c+R_{gr}}{X_{gr}} I_2 & -\frac{1}{X_{gr}} I_2 & \frac{1}{X_{gr}} I_2 \\
-\frac{1}{X_c} I_2 & -\frac{1}{X_c} I_2 & 0_{2\times2} & 0_{2\times2} \\
0_{2\times2} & 0_{2\times2} & 0_{2\times2} & \omega_g \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
\end{bmatrix}
$$

$$
E = -\frac{V_{dc}}{2X_{lc}} \begin{bmatrix} I_2 & 0_{2\times6} \end{bmatrix}^T
$$

$$
C = \begin{bmatrix} I_6 & 0_{6\times2} \end{bmatrix},
$$

where $R_{lc}$, $X_{lc}$, $R_{gr}$, $X_{gr}$, $R_c$ and $X_c$ are system parameters in per unit; $R_{lc}$ ($X_{lc}$) is the converter-side LCL filter resistance (reactance). $R_{gr}$ ($X_{gr}$) is the equivalent grid-side resistance (reactance), and $X_c$ and $R_c$ are the filter capacitance and its parasitic resistance, respectively. $\omega_g$ is the angular grid frequency and $0$ is the zero matrix of denoted dimensions. The values of the system parameters including the dc-link voltage, the converter and the LCL filter parameters are given in Table 2.2.

The $LCL$ filter has two resonance frequencies of which the higher one is the main and it can be calculated as

$$
f_{res} = \frac{1}{2\pi} \sqrt{\frac{1}{
\frac{L_cC(L_{tg}+L_d)}{L_c+L_{tg}+L_d}} \approx 1203.3\text{Hz}.}
$$

The discrete-time state-space model of the system can be derived according to 2.2.

The rated grid parameters are given in Table 2.1. Ordinarily, the grid can be described by three-phase grid voltage $v_g$, the grid resistance $R_g$, and the grid reactance $X_g$ as a more precise representation usually does not exist [9].
2.1. Case Study

The point of common coupling (PCC) in the system is where the grid is connected to the LCL filter as seen in Figure 2.2. In case of a three-phase fault at the PCC, the short-circuit power can be defined as

\[
S_{sc} = 3\left(\frac{V_g}{\sqrt{3}}\right)^2 / |Z_g| = \frac{V_g^2}{|Z_g|} \tag{2.5}
\]

where \(V_g\) denotes the line-to-line root mean square (rms) grid voltage and \(Z_g = j\omega_g L_g + R_g\), the grid impedance. Since the three-phase fault yields the maximum short circuit current, \(S_{sc}\) can be interpreted as the maximum power provided to the PCC.

The short-circuit ratio

\[
k_{sc} = S_{sc} / S_{nom} \tag{2.6}
\]

is presented which characterizes the strength of the grid. \(S_{nom}\) denotes the rated power of the converter. A strong grid is indicated by ratios above 20, with the short circuit power being large compared to the rated power of the converter. Ratios below 8 refer to a weak grid, in which the impedance of the converter is dominated by the grid impedance [3]. In general, weak grids have less stability margin and therefore the harmonics that the converter may inject into the PCC should be bounded by tighter limits [18]. Another characteristic quantity of the grid is the grid impedance ratio

\[
k_{X/R} = X_g / R_g \tag{2.7}
\]

between the grid reactance \(X_g = \omega_g L_g\) and the grid resistance \(R_g\).

Based on the grid voltage, converter power, short-circuit ratio, and grid impedance ratio given in Table 2.1, the grid inductance and resistance can be computed using equations (2.3)-(2.7) as seen in (2.8).

\[
L_g = \frac{|Z_g|}{\omega_g \sqrt{1 + 1/k_{XR}^2}} = 2.016 \text{ mH} \quad \text{and} \quad R_g = \frac{|Z_g|}{\sqrt{1 + k_{XR}^2}} = 90.509 \text{ m\Omega} \tag{2.8}
\]

<table>
<thead>
<tr>
<th>Key</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line-to-line voltage</td>
<td>400 V</td>
</tr>
<tr>
<td>Grid current</td>
<td>18 A</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Short-circuit ratio</td>
<td>20</td>
</tr>
<tr>
<td>Grid impedance ratio</td>
<td>7</td>
</tr>
</tbody>
</table>

**Table 2.1 Rated values of the grid**
2.2. Conventional Control Strategies

Several control strategies have been proposed for controlling power electronic converters and drives. Linear proportional integral (PI) and proportional resonant (PR) control, hysteresis control, deadbeat control and predictive control are among the most established control schemes in literature and also in practice [19]. Figure 2.5 shows the conventional linear PI control scheme for a grid-connected converter. In most state-of-the-art control strategies, state-space averaging method is used to neglect the switching behavior of the converter. Subsequently, state variables are transferred into the rotating orthogonal dq reference frame. This allows exploiting linear proportional-integral (PI) controllers. For the most part, the control problem at hand is a MIMO system. In case of using PI controllers, the MIMO system needs to be divided into multiple single-input single-output (SISO) controllers arranged in a cascaded form.

A modulation stage is then necessary to translate the voltage command of the inner control loop to switching signals applied to the semiconductor gates. The most frequently used modulation schemes are CB-PWM and space vector modulation.
2.3 Carrier-Based Pulse Width Modulation

In CB-PWM, a reference signal is compared to a triangular carrier waveform to generate switching signals that will be applied to the converter switches. This procedure is visualized in Figure 2.7. In steady-state operation, the symmetrical three-phase voltage reference signal $v_{abc}^*$ can be defined as

$$v_{abc}^*(t) = \hat{v} \begin{bmatrix} \sin(\omega_1 t) \\ \sin(\omega_1 t - \frac{2\pi}{3}) \\ \sin(\omega_1 t + \frac{2\pi}{3}) \end{bmatrix}$$ \hspace{1cm} (2.9)$$

where $\omega_1 = 2\pi f_1$ is the angular frequency and $\hat{v}$ is the amplitude of the voltage reference. $f_1$ denotes the fundamental frequency. The reference signal $v_{abc}^*$ which is produced by the controller is scaled to the modulating signal $u_{abc}^*$ and translated into the switching signal $u_{abc}$ via PWM as shown in Figure 2.7. The contents in this section are for the most part from [3, 20].

The modulation index $m$ can be defined as the magnitude of the modulating signal $u_{abc}^*$. Since

$$u_{abc}^*(t) = \frac{2}{v_{dc}} v_{abc}^*(t) = \frac{2}{v_{dc}} \hat{v} \begin{bmatrix} \sin(\omega_1 t) \\ \sin(\omega_1 t - \frac{2\pi}{3}) \\ \sin(\omega_1 t + \frac{2\pi}{3}) \end{bmatrix}$$ \hspace{1cm} (2.10)$$

the modulation index is given by

$$m = \frac{2}{v_{dc}} \hat{v} = \hat{u}.$$ \hspace{1cm} (2.11)$$

For $0 \leq m \leq 1$, i.e., the linear modulation range, each converter leg switches twice per carrier period $T_c = 1/f_c$, leading to a fixed switching frequency. For $m > 1$, CB-PWM enters overmodulation region in which the relationship between the demanded $m$ and the resulting magnitude of the fundamental component $\hat{u}$ is no longer linear. Overmodulation provides higher utilization of the available dc-link voltage and can maximize the output line-to-line voltage, but it comes at the expense of excited low-order harmonics.

In synchronous PWM, the ratio between the carrier frequency and the fundamental frequency $f_c/f_1$ is an integer value. For odd $f_c/f_1$ ratios, the generated pulse patterns are half-wave symmetrical; that is to say the second half of the pulse pattern in a
2.3. Carrier-Based Pulse Width Modulation

![Diagram of PWM]

Figure 2.6 Illustration of analog CB-PWM for phase a. The reference signal (blue) is compared to the carrier signal (red) to produce the switching signal (green).

Figure 2.7 Block diagram of PWM.

The fundamental period $T_1 = 1/f_1$ is the opposite of its first half, i.e., $u(t - \frac{T_1}{2}) = -u(t)$.

Provided that the half-wave symmetry exists in the switching pattern, if the pulse pattern of the first quarter of $T_1$ is mirrored with respect to its next quarter, quarter-wave symmetry is achieved. For a three-phase system this is made possible by aligning the phase difference between the carrier signal and the three reference signals. In order to ensure this alignment $f_c$ has to be an odd triplen integer multiple of the fundamental frequency $f_1$, i.e., $f_c = (3 + 6n)f_1$, where $n \in \mathbb{N}$. 
CB-PWM can be implemented either in analog, commonly referred to as natural sampling, or digitally, known as regular sampling. The digitally implemented CB-PWM introduces a small delay which can be compensated for by a phase offset in carrier signal. To achieve quarter-wave symmetric pulse patterns in analog CB-PWM, the phase shift between the carrier and the reference signals must be zero. However, the pulse pattern produced by digital CB-PWM can be quarter-wave symmetric despite a non-zero phase difference between the carrier and the reference signals. The digital sampling of CB-PWM can be done either symmetrically, i.e. either at upper or the lower peaks of the carrier signal, or asymmetrically, i.e. at both the lower and the upper peaks of the carrier signal. Asymmetric sampling causes a delay of a quarter of the carrier period, i.e., $T_c/4$.

Asymmetrically sampled synchronous CB-PWM and its resulting pulse pattern is shown in Figure 2.8. The fundamental frequency $f_1$ is 50 Hz and the carrier frequency is 450 Hz. The carrier signal has an offset of $1.5\pi f_c$, in order for the produced pulse pattern seen in Figure 2.8b to be quarter-wave symmetric.

Figure 2.9 illustrates the harmonic amplitude spectrum of the differential-mode switch position shown in Figure 2.8b. The voltage harmonics of three-phase synchronous CB-PWM are categorized as follows:

- Fundamental component: $f_{01} = f_1$
- Baseband harmonics: $f_{0\nu} = \nu f_1$ with $\nu \in \{5, 7, 11, 13, \ldots\}$
- Sideband harmonics: $f_{\mu\nu} = \mu f_c + \nu f_1$ with

$$\begin{cases} 
\mu \in \{1, 3, 5, \ldots\} \text{ and } \nu \in \{\pm 2, \pm 4, \pm 8, \pm 10, \ldots\} \\
\mu \in \{2, 4, 6, \ldots\} \text{ and } \nu \in \{\pm 1, \pm 5, \pm 7, \pm 11, \ldots\}
\end{cases}$$

As can be seen from the harmonic orders above, thanks to the quarter-wave symmetry of the pulse pattern, there are no harmonics at even multiples of the fundamental frequency. Moreover, as will be shown in the next section, in a balanced three-phase system also triplen harmonics do not exist.

### 2.3.1 Common-Mode Injection

For a balanced three-phase system the common-mode voltage is zero, i.e.,

$$v_0 = \frac{1}{3}(v_a + v_b + v_c) = 0.$$  \hfill (2.12)
2.3. Carrier-Based Pulse Width Modulation

(a) Illustration of asymmetrically sampled single-phase CB-PWM. The solid blue line indicates the reference signal and the dashed blue line shows the sampled reference signal. The carrier signal is indicated with the red line.

(b) Quarter and halfwave symmetric CB-PWM pulse pattern

Figure 2.8 Asymmetrically sampled CB-PWM and the resulting pulse pattern.
Therefore, harmonics at odd triplen multiples of the fundamental frequency do not exist. However, adding an appropriate common-mode term to the three-phase modulating signal can be effective in a sense that it allows for larger utilization of the available dc-link voltage. Using common-mode injection the modulation index can be increased as much as 15.5\% from $m = 1$ to $m = 1.155$ to enter the extended linear modulation region without going into overmodulation. It is common practice to add a common-mode term to the three-phase modulating signal. This can be done, for example, with third harmonic or min/max injection.

In third harmonic injection, to maximize the voltage boost, $1/6$ of the amplitude of the modulating signal with thrice the fundamental frequency is added to the three-phase modulating signal

$$u_0^* = \frac{m}{6} \sin(3\omega_1 t + \phi_1). \quad (2.13)$$

The addition of this term flattens the three-phase modulating signal around the peaks.

To center the three-phase modulating signal around zero, injection of the min/max term

$$u_0^* = -\frac{1}{2} (\min(u_{abc}^*) + \max(u_{abc}^*)) \quad (2.14)$$
may be employed, resulting in the same extension of the linear modulation range as with the third harmonic injection.

Injection of either common mode signals to the three-phase modulating signal as well as the resulting modulating signal is illustrated in Figure 2.10.

![Figure 2.10 Common-mode signal $u_0^*$ (dotted line) is injected to the three-phase modulating signal $u_{abc}^*$ (dash-dotted lines). The solid lines indicate the three-phase sum $u_{abc}^* + u_0^*$ of the two signals.](image)

### 2.3.2 Discontinuous Modulation

Considering switching losses is a key aspect of control design. High switching losses are undesirable in a sense that they increase the overall cost as additional components are then required to mitigate the effects of heat dissipations.

Switching frequency directly corresponds to switching losses. In order to reduce the average switching frequency, discontinuous modulation techniques can be employed [20]. In discontinuous modulation, depending on the scheme, each one of the three phases remains inactive (i.e., the corresponding switches do not change state) for specific intervals of the fundamental period. Consequently, the number of switching transitions are reduced and thus, the average switching frequency reduces.

The most straightforward approach to have discontinuous switching patterns is for the three phases to each consecutively refrain from switching for one third of the fundamental period (i.e., for $120^\circ$). In $120^\circ$ discontinuous modulation, known as DPWMMIN, each phase leg in turn is clamped to the lower dc rail for 1/3 of the modulation cycle. This reduces the average switching frequency by 33% which allows increasing the converter switching frequency by a factor of 1.5 and still obtain the
same amount of losses. The major drawback of the scheme is that the switching losses are not divided equally between the upper and lower switches in a phase leg \([20]\). Figure 2.11 illustrates the 120° discontinuous PWM scheme.

DPWM techniques can be realized by adding appropriate zero-sequence signals. In case of DPWMMIN, the three-phase modulating signal with the smallest magnitude determines the zero-sequence signal. Assume that \(u^*_a\) is the signal with the least magnitude. Then,

\[
u_0 = \left(\text{sign}(u^*_a)\right) \frac{V_{dc}}{2} - u^*_a
\]

is added to the three-phase modulating signals.

\[
\text{Figure 2.11 120° discontinuous PWM. The zero-sequence signal (grey) is added to the three-phase sinusoidal reference signal (dash dotted lines) to produce the new three-phase reference signal (solid lines).}
\]

Due to the different switching pattern, the harmonic content may increase compared to the continuous modulation scheme. It is the trade-off between the harmonic content and the switching frequency that provides advantages to exploit discontinuous modulation schemes. Figure 2.12 shows the frequency components of the differential-mode switch position produced by DPWMMIN with \(f_c/f_1 = 9\). As seen in the figure, harmonics at even multiples of the fundamental frequency appear since quarter and halfwave symmetries no longer exist \([20]\).
2.4 Optimization

In this section basic concepts of mathematical optimization are discussed. To begin with, definitions of convex set, convex function and affine function are introduced in (2.10), (2.17) and (2.18) respectively. The contents in this section are directly from [22].

A set $C$ is convex if the line segment between any two points in $C$ lies in $C$, i.e., if for any $x_1, x_2 \in C$ and any $\theta$ with $0 \leq \theta \leq 1$ we have

$$\theta x_1 + (1 - \theta)x_2 \in C. \quad (2.16)$$

A function $f: \mathbb{R}^n \to \mathbb{R}$ is convex if $\text{dom} \ f$ is a convex set and if for all $x, y \in \text{dom} \ f$, and $\theta$ with $0 \leq \theta \leq 1$, we have

$$f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y). \quad (2.17)$$

$\text{dom}$ denotes the domain (of a function). A function $f: \mathbb{R}^n \to \mathbb{R}^m$ is affine if it is the sum of a linear function plus and a constant, i.e., if it has the form

$$f(x) = Ax + b, \quad (2.18)$$
2.4. Optimization

where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

An optimization problem is of the general form

$$\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad g_i(x) \leq 0, \quad i = 1, \ldots, m \\
& \quad h_j(x) = 0, \quad j = 1, \ldots, n.
\end{align*}$$

(2.19)

where $x \in \mathbb{R}^n$ is the optimization variable which minimizes $f : \mathbb{R}^n \to \mathbb{R}$, i.e., the objective function, $g_i : \mathbb{R}^n \to \mathbb{R}$ are the inequality constraint functions and $h_j : \mathbb{R}^n \to \mathbb{R}$ are the equality constraint functions.

The intersection of domains $f, g$ and $h$ define the domain of the optimization problem, i.e.,

$$\mathcal{D} = \text{dom } f \cap \bigcap_{i=1}^{m} \text{dom } g_i \cap \bigcap_{j=1}^{p} \text{dom } h_j.$$  

(2.20)

A point $x \in \mathcal{D}$ is feasible if it satisfies all the constraints $g_i(x) \leq 0, i = 1, \ldots, m$ and $h_j(x) = 0, j = 1, \ldots, p$. A feasible set $\mathcal{F}$ is a set containing all feasible points. The optimization problem is feasible if $\mathcal{F} \neq \emptyset$ and infeasible if $\mathcal{F} = \emptyset$, where $\emptyset$ denotes the empty set.

The optimal value $q^*$ is defined as

$$q^* = \inf \{ f(x) \mid g_i(x) \leq 0, i = 1, \ldots, m, h_j(x) = 0, j = 1, \ldots, p \}$$

(2.21)

The point $x^*$ is said to be the optimal point if $x^*$ is feasible and $f(x^*) = q^*$. A set containing all optimal points is the optimal set $X_{\text{opt}}$ and is defined as

$$X_{\text{opt}} = \inf \{ x \mid f(x) = q^*, g_i(x) \leq 0, i = 1, \ldots, m, h_j(x) = 0, j = 1, \ldots, p \}$$

(2.22)

Depending on the problem, $X_{\text{opt}}$ can have any number of elements. The problem is solvable if $X_{\text{opt}} \neq \emptyset$.

2.4.1 Convex Optimization

Convex optimization is a subset of mathematical optimization. There are many algorithms for solving convex problems efficiently. A convex problem is generally of
the form [22]

\[
\text{minimize } \quad f(x) \\
\text{subject to } \quad g_i(x) \leq 0, \quad i = 1, \ldots, m \quad (2.23)
\]

\[a_j^T x = b_j, \quad j = 1, \ldots, p,
\]

where \( f(x) \) and \( g_i(x) \leq 0 \) are convex and the equality constraint functions \( a_j^T x = b_j \) are affine. This ensures that \( \mathcal{D} \) is a convex set.

### 2.4.2 Quadratic Optimization

As will be shown in Chapter 3, the optimization problem studied in this thesis can be formulated as a convex quadratic program (QP). A quadratic program is of the form

\[
\text{minimize } \quad \frac{1}{2} x^T Q x + p^T x \\
\text{subject to } \quad G x \leq h \\
A x = b,
\]

where \( Q \in S^n_+ \), \( p \in \mathbb{R}^n \), \( G \in \mathbb{R}^{m \times n} \), \( h \in \mathbb{R}^m \), \( A \in \mathbb{R}^{p \times n} \) and \( b \in \mathbb{R}^p \). \( S_+ \) is defined as the set of symmetric positive semidefinite matrices. \( Q \in S^n_+ \) implies that the problem is convex and consequently solvable in polynomial time [23].

### 2.5 Model Predictive Control

MPC has recently been gaining ground as a suitable control method for power electronic converters and drives [24]. The formulation of most MPC strategies is done in discrete-time domain with constant sampling interval \( T_s \). Using the discrete-time state-space model (2.25), the future state and output of the system is obtained using the current state and input

\[
x(l + 1) = Ax(l) + B \bar{K} u_{abc}(l) \\
y(l + 1) = C x(l + 1). \quad (2.25)
\]

The model (2.25) is used to predict the evolution of the system over a finite number of steps. The prediction horizon is defined as the time window within which the evolution of the state and output variables are computed. The number of discrete time steps \( N_p \in \mathbb{N} \) within the prediction horizon implies the prediction horizon length which is equal to \( N_p T_s \).

To express the possible control actions within the prediction horizon, the sequence
of input vectors $U(k)$ is defined as

$$U(k) = \begin{bmatrix} u_{abc}^T(k) & u_{abc}^T(k+1) & \ldots & u_{abc}^T(k+N_p-1) \end{bmatrix}^T. \quad (2.26)$$

For enhanced operation of the plant an optimization problem subject to certain constraints is solved. The cost function \((2.27)\) expresses the control objectives as a function of the system states and inputs:

$$J(x(k), U(k)) = \sum_{l=k}^{k+N_p+1} \Lambda(x(l), U(l)) \quad (2.27)$$

$\Lambda(\cdot, \cdot)$ specifies the cost at each time step. The control objectives are mapped into the cost (objective) function such that a lower cost corresponds to a more desirable output. That is to say the tracking error of the output variables for each possible control action is translated into a single scalar value (i.e., cost) and the action that results in the minimum cost is selected and applied in order to ensure optimal operation of the plant.

The optimization problem is usually solved under a number of constraints. The constraint come in the forms of equalities and inequalities over any of the concerned variables.

The general form of the constrained optimization problem solved is:

$$\begin{aligned}
\text{minimize } & \quad J(x(k), U(k)) \text{ see (2.27)} \\
\text{subject to } & \quad x(l+1) = Ax(l) + Bu(l) \\
& \quad y(l+1) = Cx(l+1) \\
& \quad \forall l = k, \ldots, k + N_p - 1.
\end{aligned} \quad (2.28)$$

Solving the optimization problem yields the optimal sequence of control actions and the cost associated with that sequence. The optimal sequence of control actions $U_{opt}$ is the sequence resulting in the minimum cost while adhering to the constraints imposed.

$U_{opt}(k)$ is the open-loop solution to the control problem at time step $k$. To achieve closed-loop control, only the first element of $U_{opt}(k)$ is applied at time-step $k$. At the next time step, the prediction horizon is shifted by one step and the optimization problem is solved based on new measurements/estimates. This is called the receding horizon policy \([25]\). In other words, the variables are measured or estimated at each time-step and the optimization problem is solved using these measured/observed
2.5. Model Predictive Control

![Diagram of prediction horizon at time step k and k+1]

(a) Prediction horizon at time step $k$

(b) Prediction horizon at time step $k+1$

**Figure 2.13** Receding horizon policy visualization for $N_p = 4$. The solution for the optimization problem yields the sequence of manipulated variables $U_{opt}$ which makes the predicted output sequence $\hat{Y}$ track the output reference $Y^\ast$. Only the first element of $U_{opt}$, i.e., $u_{abc, opt}$ is applied at each time step.

values. Therefore, the performance of the controller is not significantly affected even if few modeling mismatches occur. Since the prediction horizon is shifted one step forward at each new time-step, the length of the prediction horizon stays constant. Figure 2.13 illustrates the notion of receding horizon policy for $N_p = 4$.

Although only the first element of $U_{opt}$ is applied at each time step, longer prediction horizons are deemed beneficial. This is due to the fact that the behavior of the
2.5. Model Predictive Control

system in the whole time window is considered when solving the optimization problem. Therefore, a longer prediction horizon means that the behavior of the system is anticipated further into the future. Consequently, better educated guesses can be made when having longer prediction horizons [3].

In [27], it is shown that longer horizons provide performance benefits in MPC for power electronics. Nevertheless, the computational complexity of the problem may increase as $N_p$ increases.

MPC can be summarized in the following steps:

1. Measurement/estimation of the state variables
2. Prediction of the system behavior over the prediction horizon
3. Solving the optimization problem and obtaining the optimal control actions
4. Applying the optimal control actions for the first time step
5. Shifting the prediction horizon one step forward
6. Repeat.
3. DIRECT MODEL PREDICTIVE CONTROL
WITH FIXED SWITCHING FREQUENCY

In this chapter, direct MPC with constant switching frequency is introduced. Aptly
named, the proposed method yields a constant switching frequency despite the fact
that the switches are controlled directly, i.e., the absence of a modulator. This is
done by switching each phase once per sampling interval.

Direct MPC with fixed switching frequency algorithm consists of two main parts
\cite{26,14}:

1. Calculating slopes (gradients) of the controlled variables
2. Solving the optimization problem

3.1 Definition and Constraints

For a two-level inverter, the average switching frequency $f_{sw}$ can be obtained by

$$f_{sw} = \lim_{N \to \infty} \frac{1}{2} \cdot \frac{1}{6N T_s} \sum_{l=0}^{N-1} \|u_{abc}(l) - u_{abc}(l-1)\|_1,$$  \hspace{1cm} (3.1)

where $N \in \mathbb{N}$ denotes the number of sampling intervals, $6$ stands for the number of
active switches and $\| \cdot \|_1$ represents the 1-norm.

To obtain constant switching frequency, each phase leg must switch once per sam-
pling interval. This fact can be indicated as

$$\|u_{abc}(k + 1) - u_{abc}(k)\|_1 = 6.$$  \hspace{1cm} (3.2)

With this constraint, the equation 3.1 simplifies to $f_{sw} = \frac{1}{2 T_s}$.

Another restriction is that two phase legs cannot switch simultaneously. Keeping
the constraints above in mind, it can be concluded that a total of four different
3.1. Definition and Constraints

<table>
<thead>
<tr>
<th>Switching sequence</th>
<th>Switch positions</th>
</tr>
</thead>
<tbody>
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<td>$a \to b \to c$</td>
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</tr>
<tr>
<td>$a \to c \to b$</td>
<td>$-1 \to -1 \to -1 \to 1$</td>
</tr>
<tr>
<td>$b \to a \to c$</td>
<td>$-1 \to -1 \to 1 \to 1$</td>
</tr>
<tr>
<td>$b \to c \to a$</td>
<td>$-1 \to -1 \to -1 \to 1$</td>
</tr>
<tr>
<td>$c \to a \to b$</td>
<td>$-1 \to -1 \to -1 \to 1$</td>
</tr>
<tr>
<td>$c \to b \to a$</td>
<td>$-1 \to -1 \to 1 \to 1$</td>
</tr>
</tbody>
</table>

Table 3.1 Possible switching sequences/positions for $u_{abc}(k) = [-1 \ -1 \ -1]^T$

Switch positions are applied to the converter switches within each sampling interval. For instance, let $u_{abc}(k) = [-1 \ -1 \ -1]^T$. Then at time step $k+1$ the switch position will be $u_{abc}(k+1) = [1 \ 1 \ 1]^T$. Since the phases are supposed to switch once at a time, if phase $a$ switches first, $b$ second and $c$ last, it follows that the switching is done in the order: $[-1 \ -1 \ -1]^T \to [1 \ -1 \ -1]^T \to [1 \ 1 \ -1]^T \to [1 \ 1 \ 1]^T$. As the switch position at time step $k+1$ is $u_{abc}(k+1) = [1 \ 1 \ 1]^T$, it can be concluded that the switch position at time step $k+2$ returns to $u_{abc}(k+2) = [-1 \ -1 \ -1]^T$.

At the beginning of a sampling interval ($t_0$), the switch position applied at the end of the previous sampling interval is still being applied. Thus, we define times $t_1$, $t_2$ and $t_3$ within each sampling interval as the instants that the switching actions take place in three phases. To be more specific, these instants can be defined as:

1. $t_1 \in (0, T_s)$: the time instant that the first phase leg switches
2. $t_2 \in (0, T_s)$: the time instant that the second phase leg switches
3. $t_3 \in (0, T_s)$: the time instant that the third phase leg switches.

These switching instants occur successively within one sampling interval. Therefore,
the following inequality holds true:

\[ 0 < t_1 < t_2 < t_3 < T_s. \] (3.3)

We define the vector of switching instants as

\[ \mathbf{t} = [t_1 \ t_2 \ t_3]^T. \] (3.4)

Moreover, the vector of switch positions (the switching sequence) \( \mathbf{U} \) is defined as

\[ \mathbf{U} = [\mathbf{u}_{abc}^T(t_0) \ \mathbf{u}_{abc}^T(t_1) \ \mathbf{u}_{abc}^T(t_2) \ \mathbf{u}_{abc}^T(t_3)]^T. \] (3.5)

Depending on the order the phase legs switch, there can be 6 different switching sequences. Assuming that the switching starts from \([-1 -1 -1]^T\) at time step \( k \), the possible switching sequences and the resulting switching positions are given in Table 3.1

### 3.1.1 Slopes of the Controlled Variables

The discrete-time model (2.25) allows predicting the evolution of the controlled variables over \( T_s \) based on the applied switch position at the beginning of the sampling interval. If we assume that the dynamic evolution of the controlled variables is linear within the sampling interval, the prediction can be addressed based on the slopes of the output variables. The premise is reasonable since we assume \( T_s \) to be very small. Calculating the slopes is the prelude to finding the optimal switching sequence and instants to be applied to the converter.

As mentioned earlier, regardless of the switching sequence, there are three switching instants in each sampling interval. This divides \( T_s \) into four subintervals as seen in Figure 3.1. The slopes of the controlled variables at each subinterval can be calculated based on the continuous-time state state-space representation introduced in (2.1) as

\[ \mathbf{m}(t_z) = \frac{\mathbf{d}y(t_z)}{dt} = C\frac{\mathbf{d}x(t_z)}{dt} = C(D\mathbf{x}(t_0) + E\mathbf{u}_{abc}(t_z)) \] (3.6)

where \( z \in \{0, 1, 2, 3\} \). To calculate the slopes, we use the state at \( t_0 \) at all subintervals since we assume the gradients to be constant within the sampling interval.
3.1. Definition and Constraints

3.1.2 Objective Function

The objective of the proposed control method is to minimize the ripple in the controlled variables with respect to their references while maintaining a fixed switching frequency. MPC is formulated as an optimization problem which has an objective function. The control objectives should be mapped into the objective function of the optimization problem. The ripple in the controlled variables can be introduced in the form of squared rms error as

$$ e_{rms} = \frac{1}{T_s} \int_0^{T_s} (y^*(t) - y(t))^2 \, dt $$

where $y^* \in \mathbb{R}^{n_y}$ is the vector of the references quantities of the controlled variables. Since it is sometimes required to give the minimization of the error for some of the controlled variables a higher priority than the rest, a weighting matrix can be employed in the squared rms error as

$$ e_{rms} = \frac{1}{T_s} \left( \int_0^{T_s} (y^*(t) - y(t))^T Q (y^*(t) - y(t)) \, dt \right) $$

where $Q \in \mathbb{R}^{n_y \times n_y}$ is a diagonal positive definite matrix whose entries define the prioritization of the tracking accuracy among the different controlled variables.
3.1. Definition and Constraints

The weighted squared rms error (3.8) can be rewritten in the form

\[ e_{\text{rms}} = \frac{1}{T_s} \left( \int_0^{T_s} \| y^*(t) - y(t) \|^2 \, dt \right). \]  

(3.9)

The objective function can be equal to this error. However, the minimization of the rms error for this problem may lead to non-convexity. Therefore, instead of the rms error, we choose to penalize the sampled squared error only at time instants \( t_1, t_2, t_3 \) and \( T_s \). Hence, the objective function becomes

\[ J = \| y^* - y(t_1) \|_Q^2 + \| y^* - y(t_2) \|_Q^2 + \| y^* - y(t_3) \|_Q^2 + \| y^* - y(T_s) \|_Q^2 \]  

(3.10)

where \( y^* = y^*(t_0) \). The values of the controlled variables at time instants \( t_1, t_2, t_3 \) and \( T_s \) are calculated according to Figure 3.1 as

\begin{align*}
    y(t_1) &= y(t_0) + m(t_0) \cdot t_1 \quad \text{(3.11a)} \\
    y(t_2) &= y(t_1) + m(t_1) \cdot (t_2 - t_1) \\
    &= y(t_0) + (m(t_0) - m(t_1)) \cdot t_1 + m(t_1) \cdot t_2 \quad \text{(3.11b)} \\
    y(t_3) &= y(t_2) + m(t_2) \cdot (t_3 - t_2) \\
    &= y(t_0) + (m(t_0) - m(t_1)) \cdot t_1 + (m(t_1) - m(t_2)) \cdot t_2 + m(t_2) \cdot t_3 \quad \text{(3.11c)} \\
    y(T_s) &= y(t_3) + m(t_3) \cdot (T_s - t_3) \\
    &= y(t_0) + (m(t_0) - m(t_1)) \cdot t_1 + (m(t_1) - m(t_2)) \cdot t_2 \\
    &\quad + (m(t_2) - m(t_3)) \cdot t_3 + m(t_3) \cdot T_s. \quad \text{(3.11d)}
\end{align*}

Function 3.10 can be rewritten in vector form as

\[ J = \left\| \begin{bmatrix} y^* - y(t_1) \\ y^* - y(t_2) \\ y^* - y(t_3) \\ y^* - y(T_s) \end{bmatrix} \right\|_Q^2 = \| r - M t \|_Q^2 \]  

(3.12)

where

\[ r = \begin{bmatrix} y^* - y(t_0) \\ y^* - y(t_0) \\ y^* - y(t_0) \\ y^* - y(t_0) - m(t_3) T_s \end{bmatrix}, \quad t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \]
and

\[
M = \begin{bmatrix}
    m(t_0) & 0 & 0 \\
    m(t_0) - m(t_1) & m(t_1) & 0 \\
    m(t_0) - m(t_1) & m(t_1) - m(t_2) & m(t_2) \\
    m(t_0) - m(t_1) & m(t_1) - m(t_2) & m(t_2) - m(t_3)
\end{bmatrix}.
\]

Moreover, \( \tilde{Q} = \text{diag}(Q, \ldots, Q) \) and \( \mathbf{0} \) is the \( n_y \)-dimensional zero vector.

### 3.1.3 Optimization

The MPC algorithm solves an optimization problem at each sampling interval. The solution of the optimization problem is the optimal switching sequence to be applied at the optimal time instants.

To begin with, the possible switch positions for the possible switching sequences introduced in Table 3.1 are enumerated based on the three-phase switch position applied at the end of the previous sampling interval. Next, the slopes of the output variables are calculated as explained in Section 3.1.1.

Thereafter, an optimization problem is solved for each given switching sequence. The optimization problem is a convex QP and can be summarized as

\[
\begin{align*}
\text{minimize} \quad & \| r - Mt \|_Q^2 \\
\text{subject to} \quad & \text{(2.23), (3.2) and (3.3)} \\
& t \in \mathbb{R}^3.
\end{align*}
\]

The solution of the QP for each of the six possible switching sequences is a distinct vector of three switching instants and the value of the objective function. The switching sequence and its corresponding vector of switching instants which have the minimum objective function value are the optimal solutions of the QP. The resulting optimal switching sequence is applied at the optimal switching instants to the converter.

The block diagram of direct MPC with fixed switching frequency is shown in Figure 3.2. The pseudo code of the proposed control algorithm is summarized in Algorithm 1.
Figure 3.2 Direct model predictive control with fixed switching frequency for two-level grid-connected inverter.

Algorithm 1 Direct MPC with fixed switching frequency

Given $u_{abc}(t_0)$, $y^*(t_0)$ and $x(t_0)$
0. Enumerate $U_z, z \in \{1, 2, \ldots, 6\}$ based on $u_{abc}(t_0)$.
1. Compute $m(t_0), m(t_1), m(t_2), m(t_3)$ for a given $U_z, z \in \{1, 2, \ldots, 6\}$.
2. Solve the QP [3.13] for each $U_z$. This results in $t_z$ and $J_z, z \in \{1, 2, \ldots, 6\}$.
3. arg(min($J_z$)), i.e., $t_{opt}$ and $U_{opt}$ is yielded.
4. Apply $U_{opt}$ at times $t_{opt}$.
Repeat.

3.2 Refinements

Several refinements are proposed in this section in order to improve the performance of the direct MPC scheme. These refinements can be combined together in order to achieve improved results. These improvements mainly affect the rather low-frequency harmonics in the grid currents as will be shown in Chapter 4.
3.2.1 Linear Approximation of the Reference Quantities

Thus far, the reference quantities of the output variables were assumed to be constant within the sampling interval as seen in Figure [3.1]. This is a rough approximation which leads to tracking inaccuracy since the reference quantities vary sinusoidally in time.

To have a better approximation, the evolution of the reference quantities can be linearly interpolated between the current and the next sampling interval, i.e., discrete time steps \(k\) and \(k+1\) as

\[
y^*(t) = y^*(k) + \frac{y^*(k+1) - y^*(k)}{T_s} t
\]

for \(t \in [0, T_s]\). With this refinement, the dimensions of \(t\) and \(r\) stay unchanged. However \(M\) takes the form

\[
M = \begin{bmatrix}
m(t_0) - m_i(k) & 0 & 0 \\
m(t_0) - m(t_1) & m(t_1) - m_i(k) & 0 \\
m(t_0) - m(t_1) & m(t_1) - m(t_2) & m(t_2) - m_i(k) \\
m(t_0) - m(t_1) & m(t_1) - m(t_2) & m(t_2) - m(t_3)
\end{bmatrix}
\]

where \(m_i(k) = \frac{y^*(k+1) - y^*(k)}{T_s}\). Linear approximation of the reference quantities of the controlled variables is illustrated in Figure [3.3].
Figure 3.4 Example of a prediction horizon over two sampling intervals with six switching instants.

3.2.2 Longer Prediction Horizons

Longer prediction horizons provide performance improvements to the MPC controller, especially for higher-order MIMO systems [27]. By looking further into the future, the controller makes better educated decisions. The main benefits of utilizing longer prediction horizons are the reduction of the THD of the current and the mitigation of low-frequency harmonics.

Consider a prediction horizon over two sampling intervals, as shown in Figure 3.4. The vector of switching times and switching sequences in the $k$th sampling interval which corresponds to $t \in [0, T_s)$ can be defined as

\begin{align}
    t(k) &= \begin{bmatrix} t_1(k) & t_2(k) & t_3(k) \end{bmatrix}^T \\
    U(k) &= \begin{bmatrix} u_{abc}^T(t_0(k)) & u_{abc}^T(t_1(k)) & u_{abc}^T(t_2(k)) & u_{abc}^T(t_3(k)) \end{bmatrix}^T.
\end{align}

For a prediction horizon over two sampling intervals, we define

\begin{align}
    t &= \begin{bmatrix} t(k) & t(k+1) \end{bmatrix}^T \\
    U &= \begin{bmatrix} U^T(k) & U^T(k+1) \end{bmatrix}^T.
\end{align}

where the discrete time step $k + 1$ corresponds to the continuous-time instant $T_s$.

Longer prediction horizons increase the computational complexity of the control problem as the number of possible switching sequences grows exponentially as $N_p$ is increased. To prevent this, we assume that the switching sequence of the second
sampling interval mirrors that in the first sampling interval as seen in Figure 3.4\textsuperscript{a}, i.e.,

$$U(k + 1) = [u_{abc}^T(t_3(k)) \ u_{abc}^T(t_2(k)) \ u_{abc}^T(t_1(k)) \ u_{abc}^T(t_0(k))]^T. \quad (3.17)$$

Consequently, the number of possible switching sequences is kept the same rather than being squared and thus, further computational complexity of the problem is avoided. The approach can be easily extended to even longer prediction horizons. However, higher $N_p$ values do not necessarily improve the performance of the system [28].

According to the receding horizon policy, only the first element of $U_{\text{opt}}(k)$ and $t_{\text{opt}}(k)$ are applied to the converter at time step $k$. At the next time step $k + 1$, the procedure is repeated using the new measurements of the state variables and the last applied switch position in time step $k$. This is illustrated in Figure 3.5\textsuperscript{a}.

### 3.2.3 Heavier Penalization of the Discrete Time Steps

Consider the simplified grid-connected converter with an $L$ filter shown in Figure 3.6\textsuperscript{a}. The equivalent circuit of this system is depicted in Figure 3.7\textsuperscript{b} where
3.2. Refinements

\[ L_{eq} \text{ denotes the total inductance of the filter and the grid, and the resistances are neglected. The equivalent circuit can be described by the equation } \]

\[
\frac{di_g}{dt} = \frac{1}{X_{L,eq}} (v_g - v_{conv}) \tag{3.18}
\]

where \( X_{L,eq} \) is the equivalent reactance of the circuit in per unit.

The grid voltage can be considered as a vector in the \( \alpha \beta \)-plane rotating counterclockwise with the angular speed \( \omega = 2\pi f_1 T_s \). The grid voltage vector is shown in Figure 3.8 located at an arbitrary sector of the two-level converter voltage vector in \( \alpha \beta \)-plane. Since the converter voltage corresponds to the three-phase switch position in equation (2.3) and consequently to the voltage vectors in Figure 2.3, the derivative of the grid current is proportional to the vector differences shown in Figure 3.8.

Let \( u_{abc}(t_0) = [-1 -1 -1]^T \) be the switch position being applied at time step \( k \) and \( u_{abc}(t_1), u_{abc}(t_2) \) and \( u_{abc}(t_3) \) the switch positions applied next, successively. The switch positions at time instants \( t_0, t_1, t_2 \) and \( t_3 \) correspond to the voltage vectors \( u_0, u_1, u_{i+1} \) and \( u_7 \), respectively. Assuming the grid voltage \( v_{g} \) is located at the arbitrary Sector \( l \) of the two-level converter voltage vector diagram at time step \( k \), based on the switching sequence and the corresponding voltage vectors applied, the vector differences 1, 2 and 3 yield as seen in Figure 3.8.

As the vector differences 1, 2 and 3 are proportional to the derivative of the grid current, they can be associated with the grid current ripple. If the deviation of the grid current from its reference is mapped into the \( \alpha \beta \)-plane with the \( \alpha \)-component of the grid current ripple in the \( x \)-axis and \( \beta \)-component in the \( y \)-axis, Figure 3.9 would result.
3.2. Refinements

![Equivalent circuit of the L filter equipped grid-connected converter.](image)

**Figure 3.7** Equivalent circuit of the L filter equipped grid-connected converter.

![Arbitrary Sector l of the two-Level converter voltage vector diagram.](image)

**Figure 3.8** The arbitrary Sector l of the two-Level converter voltage vector diagram. The three vector differences (dashed red) result from subtracting $v_g$ (blue) from the voltage vectors (black).

As seen in Figure [3.9] it is expected that the ripple would start at the origin at time step $k$ and return to the origin at each discrete time step $k + n, n \in \mathbb{N}$. However, since in the objective function the penalization of the error at all four time instants $t_1, t_2, t_3$ and $T_s$ has been given the same priority, the ripple tends to deviate from the origin at discrete time steps as seen in Figure [3.10].

The deviation of the controlled variables from their reference values at discrete time step $k + 1$ should be mitigated. To do this, the sampled squared error at time step $k + 1$ needs to be penalized more heavily. Thus, we introduce the weighting matrix $Q'$ to be used instead of $Q$, i.e., the weighting matrix of the objective function, for the last term in (3.10). The entries of $Q'$ must be greater than the entries of $Q$. 

With this change the objective function (3.10) transforms to

\[ J = \| \mathbf{y}^* - \mathbf{y}(t_1) \|^2_Q + \| \mathbf{y}^* - \mathbf{y}(t_2) \|^2_Q + \| \mathbf{y}^* - \mathbf{y}(t_3) \|^2_Q + \| \mathbf{y}^* - \mathbf{y}(T_s) \|^2_{Q'} \]  \quad (3.19)

Since \( Q' \) is positive definite, the equation

\[ Q' = \Gamma^T Q \Gamma \]

holds true. \( \Gamma \) can be defined as \( \Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_{n_y}) \).

Therefore, the weighted sampled squared error at the discrete time step \( k + 1 \) can be rewritten as

\[ \| \mathbf{y}^* - \mathbf{y}(T_s) \|^2_{Q'} = \| \mathbf{y}^* - \mathbf{y}(T_s) \|^2_{\Gamma^T \Gamma Q} = \| \Gamma (\mathbf{y}^* - \mathbf{y}(T_s)) \|^2_Q \]  \quad (3.20)

With this change, the objective function (3.19) can be stated in the vector form

\[ J = \left\| \begin{bmatrix} \mathbf{y}^* - \mathbf{y}(t_1) \\
\mathbf{y}^* - \mathbf{y}(t_2) \\
\mathbf{y}^* - \mathbf{y}(t_3) \\
\Gamma (\mathbf{y}^* - \mathbf{y}(T_s)) \end{bmatrix} \right\|^2_Q = \| \mathbf{r'} - \mathbf{M}' \mathbf{t} \|^2_Q \]  \quad (3.21)
Figure 3.10 The actual grid current ripple in $\alpha\beta$-plane. The end arrow of trajectory 3 shows the ripple at time step $k+1$.

where

$$r' = \begin{bmatrix} y^* - y(t_0) \\ y^* - y(t_0) \\ y^* - y(t_0) \\ \Gamma(y^* - y(t_0) - m(t_3)T_s) \end{bmatrix}$$

and

$$M' = \begin{bmatrix} m(t_0) & 0 & 0 \\ m(t_0) - m(t_1) & m(t_1) & 0 \\ m(t_0) - m(t_1) & m(t_1) - m(t_2) & m(t_2) \\ \Gamma(m(t_0) - m(t_1)) & \Gamma(m(t_1) - m(t_2)) & \Gamma(m(t_2) - m(t_3)) \end{bmatrix}.$$ 

The dimension of the vector of the switching instants remains intact. Moreover, matrix $M'$ can be altered to also include the linear approximation of the reference quantities of the controlled variables.

In this case study an LCL filter is placed between the converter and the grid instead of the simplified $L$ filter assumed in this section. Thus, the grid current ripple is affected by the characteristics of the LCL filter. Thus, the entries of $\Gamma$ in this case study should be selected such that they put the penalization of the converter
current (the unfiltered current) at a higher priority. This will accordingly affect the grid current ripple and consequently reduce the THD.

Although each of the proposed refinements contribute to improving the performance of the system in terms of current THD and tracking accuracy, to achieve the best possible results it is advised to apply all three of the proposed refinements to the algorithm.

### 3.3 Direct MPC with Fixed Switching Frequency Emulating DPWMMIN

To imitate the switching pattern of DPWMMIN, the possible switching sequences taken into account so far need to be modified. In DPWMMIN, each phase leg in turn is continuously clamped to the negative DC rail \( u = -1 \) for 120° (one-third) of the fundamental period. That is to say that the switch position \( \mathbf{u}_{abc} = [1 \ 1 \ 1]^T \) will never be applied to the converter switches. Therefore, the two-level converter voltage vectors in stationary reference frame shown in Figure 2.3 will lack the zero vector \( \mathbf{u}_z(1 \ 1 \ 1) \) and only one zero vector, i.e., \( \mathbf{u}_0(-1 \ -1 \ -1) \) will be employed.

The voltage vector diagram for the two-level converter can be divided into six 60° sections as seen in Figure 3.11. The possible switching sequence can be identified using the location of a reference vector. The reference vector \( \mathbf{v}^* \) drives the outputs to their references within the sampling interval \( T_s \) and is calculated in a deadbeat fashion.

To calculate the reference vector, we first apply the Kirchhoff’s voltage law (KVL) to the equivalent circuit of the case study shown in Figure 2.4. The purpose of calculating the reference vector is to find the output voltage of the converter which drives the converter current to its reference. Thus, we obtain

\[
\mathbf{v}^*(k) = -(R_{lc} + R_c)i_{conv}(k) - X_{lc} \frac{di_{conv}(k)}{dt} + \mathbf{v}_c(k).
\]

The discrete time approximation of the mathematical relationship above yields

\[
i_{conv}(k+1) - i_{conv}(k) = \frac{T_s}{X_{lc}} (\mathbf{v}_c(k) - \mathbf{v}^*(k) - (R_{lc} + R_c)i_{conv}(k)).
\]

We assume that the converter current at time step \( k + 1 \) is equal to its reference, i.e., \( i_{conv}(k+1) = i^*_{conv}(k+1) \). Thus, the reference vector can be calculated by

\[
\mathbf{v}^*(k) = \mathbf{v}_c(k) - (R_{lc} + R_c)i_{conv}(k) - \frac{X_{lc}}{T_s} (i^*_{conv}(k+1) - i_{conv}(k)). \quad (3.22)
\]
Figure 3.11 Two-level converter voltage vector diagram based on DPWMMIN.

Table 3.2 Possible switching sequences for $u_{abc}(k) = [-1 -1 -1]^T$

<table>
<thead>
<tr>
<th>Location of the reference vector</th>
<th>Possible switching sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sectors 1 and 2</td>
<td>$a \rightarrow b$ or $b \rightarrow a$</td>
</tr>
<tr>
<td>Sectors 3 and 4</td>
<td>$b \rightarrow c$ or $c \rightarrow b$</td>
</tr>
<tr>
<td>Sectors 5 and 6</td>
<td>$a \rightarrow c$ or $c \rightarrow a$</td>
</tr>
</tbody>
</table>

When the reference vector is located at sectors 1 and 2, phase leg $c$ is clamped to the $u = -1$ and only phase legs $a$ and $b$ can switch. At sectors 3 and 4, phase leg $a$ is clamped and the phase legs $b$ and $c$ can switch. Finally, at sectors 5 and 6, phase leg $b$ is clamped and the remaining two phase legs may switch.

The location of the reference vector is observed at each sampling interval and the possible switching sequences are generated. If the reference vector is located at sector 1, there are two possible switching sequences; either phase leg $a$ switches first and $b$ next, or vice versa. Assuming $u_{abc}(k) = [-1 -1 -1]^T$, the possible switching sequences for each sector are summarized in Table 3.2.

Thus, when emulating DPWMMIN with direct MPC with fixed switching frequency, only two of the three phase legs switch within a sampling interval. Therefore, it is only necessary to find two optimal switching instants in each sampling interval.
Consequently, the dimension of the optimization problem changes. The basic form
of the objective function can be stated as

\[
J = \left\| \begin{bmatrix} y^* - y(t_1) \\ y^* - y(t_2) \\ y^* - y(T_s) \end{bmatrix} \right\|^2 = \| r - Mt \|^2 \tag{3.23}
\]

where

\[
r = \begin{bmatrix} y^* - y(t_0) \\ y^* - y(t_0) \\ y^* - y(t_0) - m(t_2)T_s \end{bmatrix}, \quad M = \begin{bmatrix} m(t_0) & 0 \\ m(t_0) - m(t_1) & m(t_1) \\ m(t_0) - m(t_1) & m(t_1) - m(t_2) \end{bmatrix}
\]

and \( t = [t_1 \ t_2]^T \).

The MPC algorithm then chooses the optimal switching sequence and switching
times which are applied to the switches as explained in Section 3.1.3.

All of the refinements introduced in Section 3.2 are also applicable to the emulation
of discontinuous modulation in MPC and can be implemented accordingly.
4. SIMULATION RESULTS AND ANALYSIS

In Chapter 3, direct MPC with fixed switching frequency was introduced. In Section 3.3, emulation of DPWMMIN with direct MPC with fixed switching frequency was explained. The performance of the two schemes is studied using simulations carried out with MATLAB m-files and the obtained harmonic content during steady-state operation is compared to open-loop CB-PWM with third harmonic injection and DPWMMIN.

The three-phase two-level grid-connected converter with a constant dc link introduced in Chapter 2 is controlled. An $LCL$ filter is used between the converter and the grid to attenuate the harmonics that the converter produces.

This chapter comprises of three sections: Section 4.1 discusses the steady-state performance of direct MPC with fixed switching frequency in both continuous and discontinuous switching modes under nominal steady-state operation and carries out a comparison on the harmonic content achieved by MPC, open-loop CB-PWM with third harmonic injection and DPWMMIN. In Section 4.2, the performance of the system is investigated during step changes in active and reactive power. Finally, in Section 4.3, the THDs obtained using the four approaches are compared to each other.

4.1 Steady-State Performance

The operating point under which the performance of the system is studied corresponds to power factor equal to 1 ($P = 1$ p.u. and $Q = 0$ p.u.). The three-phase grid current, converter current, capacitor voltage, active and reactive power, switch positions and grid current harmonic spectrum are displayed for continuous and discontinuous MPC schemes. For open-loop CB-PWM with third harmonic injection, the reference waveforms, three-phase switch positions, frequency components of $u_\alpha$ and grid current harmonic spectrum are shown.

The steady-state performance has been studied primarily for a constant switching frequency of 2850Hz. This corresponds to the carrier frequency to fundamental fre-
4.1. Steady-State Performance

Frequency ratio \( \frac{f}{f_1} \) of 57 for asymmetrically sampled CB-PWM and a sampling interval \( T_s \) of \( \frac{1}{22850 \text{Hz}} \approx 175.43 \text{ \mu s} \) for direct MPC with fixed switching frequency. The controller to system sampling time ratio is 1200 for CB-PWM and 400 for direct MPC with fixed switching frequency. Moreover, it is straightforward to extend the study to higher or lower switching frequencies. However, the tracking accuracy of the controlled variables as well as the harmonic content produced change as different switching frequencies are chosen.

Using MATLAB, discrete Fourier transform (DFT) is performed to determine the harmonic components of the three-phase grid current \( (I_h) \) and then the THD for each phase is computed up to the 200th harmonic (10 kHz) as

\[
\text{THD} = \sqrt{\sum_{h=2}^{200} I_h^2},
\]

where \( I_1 \) denotes the amplitude of the fundamental frequency component of the current [29]. The reported grid current THDs in this chapter are the average THDs of the three phases.

4.1.1 Grid Codes

The grid codes define particular guidelines for interconnection of generating units to the grid in order to ensure safe and secure operation of the system. On the grid side, these guidelines can be classified into two categories: static requirements which apply during steady-state operation, and dynamic requirements which apply during transients. Harmonic standards impose limits on harmonic emissions of current and voltage during nominal grid conditions, whereas during grid disturbances and faults the continued operation of the converter is of concern.

The standards on harmonic emissions are imposed at the PCC. The PCC is “usually taken as the point in the power system closest to the user where the system owner or operator could offer service to another user” [16]. In this case study, the PCC is shown in Figure 2.2.

Several standards proposing recommended harmonic emission limits for voltage and current exist for industrial power electronics applications. In this thesis, the IEEE 519 standard on harmonic control in power systems is considered [16].

Table 2 of IEEE 519 standard defines the current distortion limits for systems rated between 120V and 69kV. The maximum limits of the harmonic distortions are given.
as a percentage of the maximum fundamental frequency component of the current at PCC. These limits vary with the harmonic order and the short circuit ratio $k_{sc}$. As seen in Figure 4.1, the limits are tighter on harmonics of higher orders and harmonics at even multiples of the fundamental frequency. Moreover, for smaller short-circuit ratios (weaker grids), stricter limits are imposed on harmonic distortions of the current, as mentioned in Chapter 2.

In the case study investigated in this thesis, the short-circuit ratio is assumed to be equal to 20. The IEEE 519 limits on harmonic distortions for this ratio and rated voltage are shown in Figure 4.1. To take necessary precautions, it is recommended to consider the tighter limits of $k_{sc} < 20$ in industrial implementations.

4.1.2 LCL Filter

$LCL$ filters are third-order systems with a resonance peak frequency. They attenuate the harmonics appearing beyond the resonance frequency (switching harmonics) at a rate of -60dB/dec and thus are preferred over the $L$ filters in grid-connected applications. However, because of the zero impedance at the resonance frequency, harmonics neighboring $f_{res}$ get excited. To attenuate these harmonics, it is necessary to damp the undesirable resonance effects (instability and oscillations) by lowering the resonance peak. This is done by passive and active damping methods [30].

The frequency response of the transfer function of the $LCL$ filter considered in this case study is shown in Figure 4.2. As seen in the figure and (2.4), the resonance frequency of the filter is 1203.3Hz.
4.1. Steady-State Performance

4.1.3 Open-Loop Theoretical CB-PWM

Asymmetrically sampled CB-PWM with third harmonic injection is used, as seen in Figure 4.3a. Third harmonic injection into the reference is beneficial as it achieves lower harmonic distortions compared to a simple sinusoidal reference and as mentioned in Chapter 2, it increases the linear modulation range [20]. Figure 4.3b shows the steady-state three-phase switch position obtained by the scheme and Figure 4.3c the frequency components of the common-mode voltage $u_a$.

The modulation index $m = 1$ is chosen for simulation. The LCL filter attenuates the harmonic emissions above the resonance frequency and reduces the grid current THD to 0.68%. This is the achievable minimum THD one can get with CB-PWM with third harmonic injection in theory, i.e., when a controller with low bandwidth is assumed for this case study.

Figure 4.3d shows the harmonic spectrum of the grid current obtained by open-loop CB-PWM with third harmonic injection. The most prominent harmonics are at
250Hz, 350Hz, 1150Hz, 2650Hz, 2750Hz, 2950Hz and 3050Hz (5th, 7th, 23rd, 53rd, 55th, 59th and 61st, respectively) which are all odd non-triplen integer multiples of the fundamental frequency as expected due to the quarter-wave symmetry of the switching sequence. It can be concluded by comparing the figure to Figure 4.1 that the grid current harmonics produced using this scheme for $f_c/f_1 = 57$ comply with the IEEE 519 standard.

4.1.4 Direct MPC with Fixed Switching Frequency

As explained in Chapter 3, in direct MPC with fixed switching frequency just like in CB-PWM in linear modulation region, each phase leg switches once per sampling interval, with the aim to get similar switching pattern as that of CB-PWM.
4.1. Steady-State Performance

The algorithm is simulated with a two-step prediction horizon. The references are approximated linearly as explained in Section 3.2.1. The weighting matrix $\tilde{Q} = \text{diag}(1, 1, 9, 0.93, 0.93)$ is chosen for the objective function (3.13). The priority is given to the grid current. The weighting matrix of the discrete time steps defined in Section 3.2.3 is chosen as $\tilde{Q}' = \text{diag}(10.5, 10.5, 10, 10, 10)$, giving the penalization of the converter currents at discrete time steps higher priority.

The tuning process of the weighting matrices $\tilde{Q}$ and $\tilde{Q}'$ begins with initially assuming $\tilde{Q} = \tilde{Q}' = \text{diag}(1, 1, 1, 1, 1)$. We keep $\tilde{Q}'$ as is and start the tuning from $\tilde{Q}$ by first increasing the terms associated with the grid current (third and fourth terms) to an extent where the three-phase grid current follow their references as tightly as possible and thus, the grid current THD is minimized. Then, the terms associated with the capacitor voltage (fifth and sixth terms) are modified to eliminate any present steady-state error in active and reactive power. Next, the grid current ripple in $\alpha\beta$-plane is plotted and the entries of $\tilde{Q}'$ are increased so that the ripple returns to the origin at discrete time steps. This procedure results in minimum THD and good tracking of the references for the controlled variables.

As seen in Figures 4.4a and 4.4b, the LCL filter reduces the harmonic distortions of the converter current and delivers a rather smooth current to the grid. Figure 4.5 shows the obtained harmonic spectrum of the grid current by MPC. The THD of the grid current is 0.69% (only 0.01% above the theoretical minimum achieved by open-loop CB-PWM). Additionally, capacitor voltages track their references accurately as seen in Figure 4.4c. Figure 4.4d shows the three-phase switch position of the direct MPC with fixed switching frequency.

Most of the harmonics produced with direct MPC with fixed switching frequency are at the same frequencies as for CB-PWM with third harmonic injection. The most pronounced harmonics are at 250Hz, 350Hz, 2650Hz, 2750Hz, 2950Hz and 3050 Hz (5th, 7th, 53rd, 55th, 59th and 61st, respectively). The 7th harmonic is slightly pronounced in direct MPC compared to CB-PWM with third harmonic injection and the amplitudes of the 53rd and 55th have been interchanged. Harmonics around the resonance frequency are still present in the case of direct MPC with fixed switching frequency, but none of the harmonics violate the limitations of IEEE 519 standard.

The active and reactive power are not computed at the PCC, but rather before the grid reactance based on the grid voltage $v_g$ and current $i_g$. As can be seen in the equivalent circuit in Figure 2.3, the grid reactance and resistance have been replaced by taking into account the in series connection with the LCL filter. Consequently, the PCC is not properly defined in Figure 2.3 and the power of interest is computed at
the grid voltage source. The active and reactive power accurately track the desired 1 p.u. and 0 p.u. values, respectively, with no steady-state errors as shown in Figures 4.4e and 4.4f respectively.

It can be concluded that direct MPC with fixed switching frequency manages to track all of the defined reference values in steady-state operation successfully.

### 4.1.5 Open-Loop Theoretical Discontinuous CB-PWM

With asymmetrically sampled 120° discontinuous CB-PWM, it is expected to have switching frequency of $\frac{2}{3} \times 2850 = 1900$ Hz, i.e., the switching frequency decreases by 33%. This is due to the fact that we use the same carrier as in CB-PWM with continuous modulation. However, as discussed in Chapter 2, the harmonic content achieved with this approach increases.

Asymmetrically sampled 120° discontinuous CB-PWM for $f_c/f_1 = 57$ is illustrated in Figure 4.6a. The resulting three-phase switch position and the frequency components of the $a$-component of switch position are shown in Figures 4.6b, 4.6c respectively.

Figure 4.6d shows the grid current harmonic spectrum obtained by this scheme. The most prominent harmonics are at 1100Hz, 1150Hz, 1250Hz, 2650Hz, 2750Hz, 2800Hz, 2900Hz, 2950Hz and 3050Hz (22nd, 23rd, 25th, 53rd, 55th, 56th, 58th, 59th and 61st, respectively). DPWMMIN also gives rise to even order low frequency harmonics (e.g., 2nd and 4th). The THD achieved by DPWMMIN at $f_{sw} = 1900$Hz is 0.87%. The amplitude of the produced harmonics of the grid current are under the IEEE 519 standard limits as seen in Figures 4.6d and 4.11

### 4.1.6 Direct MPC Emulating DPWMMIN

DPWMMIN switching pattern can be emulated by direct MPC with fixed switching frequency as explained in Section 3.3. The algorithm is simulated for a two-step prediction horizon and the reference quantities are linearly interpolated. The weighting matrix of the objective function is chosen as $\tilde{Q} = \text{diag}(1, 1, 9, 9, 1, 1, 1)$ and the weighting matrix of the discrete time steps as $\tilde{Q}' = \text{diag}(5.8, 5.8, 5.5, 5.5, 5.5, 5.5, 5.5)$. The criteria for the selection of $\tilde{Q}$ and $\tilde{Q}'$ entries is explained in Section 4.1.4.

As seen in Figures 4.7a, 4.7c the references are well tracked during steady-state operation for direct MPC with fixed switching frequency emulating DPWMMIN.
4.1. Steady-State Performance

Figure 4.4 Steady-state performance of direct MPC with fixed switching frequency at nominal grid condition ($pf=1$) and $f_{sw} = 2850$Hz.

Figure 4.7d shows the three-phase switch position of the MPC. Due to a phase shift of $\frac{T_s}{2}$ in references, the switch positions are also shifted compared to those in Figure 4.6b. The active and reactive power track their references accurately and there are no steady-state errors as seen in Figures 4.7e and 4.7f respectively.
Figure 4.5 Grid current harmonic spectrum produced by direct MPC with fixed switching frequency. The THD is 0.69% for \( f_{\text{sw}} = 2850 \text{Hz} \).

Figure 4.8 shows the harmonics of the grid current produced by the scheme. The most prominent harmonics are of the same order as in the open-loop theoretical discontinuous CB-PWM. At the cost of slightly pronounced 4th and 8th harmonics, the harmonics around the resonance frequency obtained using DPWMMIN are attenuated. Moreover, the harmonics around 2850Hz are of the same amplitude for discontinuous direct MPC as for DPWMMIN. The THD obtained by the proposed direct MPC which emulates DPWMMIN is 0.87%, equal to the theoretical minimum achieved by DPWMMIN shown in Section 4.1.3.

4.2 Transients Performance

In this section, the performance of direct MPC with fixed switching frequency during active and reactive power transients is investigated. While the system is operating at steady-state operating point \( P = 1 \text{ p.u.} \) and \( Q = 0 \text{ p.u.} \), the active and reactive power references are stepped down to 0.5 p.u. at \( t = 100 \text{ ms} \) and stepped up to their nominal values at \( t = 300 \text{ ms} \). The grid and the converter currents as well as the capacitor voltage references are changed accordingly. The controlled variables and the powers are displayed during transients.
4.2. Transients Performance

![Graphs showing reference and carrier, three-phase switch position, frequency components of $u_\alpha$, and grid current harmonics with THD of 0.87%](image)

**Figure 4.6** Performance of 120° DPWM (DPWMMIN) at nominal grid condition ($pf=1$) and switching frequency 1900Hz.

### 4.2.1 Continuous Direct MPC

The references change can occur at any given time. Since the MPC algorithm operates in the discrete-time domain, it will react to the step changes with a delay of at most one sampling interval. Therefore, although the direct MPC algorithm is fast during transients, a slight delay in the controller response is expected.

The chosen weighting matrix for the discrete time steps is $\tilde{Q}' = \text{diag}(2.2, 2.2, 2, 2, 2, 2)$. Two weighting matrices were chosen for the objective function: for steady-state operation $\tilde{Q} = \text{diag}(1, 1, 9, 9, 0.93, 0.93)$ and for the transients $\tilde{Q} = \text{diag}(1, 1, 1.5, 1.5, 4, 4)$. A weighting matrix with different entries is used during transients to avoid any potential overshoots and to reduce the settling time. This weighting matrix is applied for a specific number of sampling intervals chosen based on trial and error. The same criteria as described in Section 4.1.4 is used for choosing the entries of $\tilde{Q}$ for
steady-state operation and thereafter, entries of $\tilde{Q}$ for the transients operation and $\tilde{Q}'$ are selected based on trial and error to minimize the overshoot based on observation from the active and reactive power plots. In case of using the same entries for $\tilde{Q}$ during steady-state and transients, the overshoot in active reactive power may reach to 1.2 p.u. 

Figure 4.7 Steady-state performance of direct MPC emulating DPWMMIN at nominal grid condition ($pf=1$) and $f_{sw} = 1900\, Hz$. 

4.2. Transients Performance
4.2. Transients Performance

Figure 4.8 Grid current harmonic spectrum produced by direct MPC with fixed switching frequency emulating DPWMMIN. The THD is 0.87% for \( f_{sw} = 1900\text{Hz} \).

As seen in Figure 4.9, the references of the controlled variables and powers are well-tracked during both before and after the step changes in the reference values. The capacitor voltages reach their peak values of 1.2 p.u. during transients as seen in Figure 4.9c. Figure 4.10 shows the zoomed-in plots of Figure 4.9 to ±10ms of the step-down and step-up instants. As seen in the figures, the controller acts fast during transients and the system reaches steady-state after 2ms which corresponds to roughly 15 sampling intervals when the switching frequency is \( f_{sw} = 2850\text{Hz} \). The transient for all of the variables lasts longer during step-down in active power than that of the step-up as seen in zoomed in figures. Slower dynamics during step-down in active power are due to the smaller voltage margin available when operating in nominal grid condition. No steady-state error is present in active and reactive power during steady-state as seen in Figures 4.9d and 4.9e. The resulting three-phase switch positions during transients are shown in Figures 4.10a and 4.10b.

4.2.2 Discontinuous Direct MPC

In this section, the performance of direct MPC with fixed switching frequency emulating DPWMMIN is investigated during power transients. When emulating DPWMMIN with MPC, one phase leg at a time is clamped to the negative dc rail for one-third of the fundamental period. Therefore, two phases at a time are actively involved in producing the desired output and the switching frequency reduces to
2/3 of that in the continuous scheme. Nevertheless, it will be shown that similar transients performance as in the continuous switching mode is achieved using the discontinuous scheme.

The chosen weighting matrix for the discrete time steps is $\tilde{Q}' = \text{diag}(3, 3, 3, 3, 3)$. 

**Figure 4.9** Transients performance of direct MPC with fixed $f_{sw}$. The power references are shifted from their nominal values to 0.5 p.u. The switching frequency is 2850Hz.
Three weighting matrices were chosen for the objective function: For steady-state operation at the first operating point $\tilde{Q} = \text{diag}(1, 1, 9, 9, 1.8, 1.8)$, for steady-state operation at the second operating point $\tilde{Q} = \text{diag}(1, 1, 11, 11, 1.1, 1.1)$, and during the transients $\tilde{Q} = \text{diag}(1, 1, 1.5, 1.5, 2.2, 2.2)$ were chosen. The criteria for choosing the entries of the weighting matrices is explained in the previous section.
4.2. Transients Performance

Figure 4.10 Transients performance of direct MPC (zoomed-in). The powers and the three-phase switch positions are shown for ±10ms of the reference change instants.

Although in Section 4.2.1 only one weighting matrix was used during steady-state operation of the two operating points, to improve the tracking accuracy of the controlled variables as well as to reduce the overshoot during transients, we opted for a separate weighting matrix for steady-state operation at each operating point.
As seen in Figure 4.11, the controlled variables as well as powers track their references accurately. The peak in capacitor voltages happens during transients and is approximately 1.2 p.u.. The ripple in active and reactive power is greater when emulating discontinuous modulation compared to continuous scheme. This is due to the fact that only two of the three phase legs are involved at a time in producing
the output voltage of the converter and thus, the ripple increases.

Figure 4.12 shows the zoomed-in plots of Figure 4.11 to ±10ms of the step-down and step-up instants. As in continuous operation, the transient lasts longer during step-down than during step-up due to the smaller voltage margin available.
Figure 4.12 Transients performance of DMPC emulating DPWMMIN (zoomed-in). The powers and the switch positions are shown for ±10ms of the reference change instants.

The three-phase switch positions during transients are shown in Figures 4.12k. [4.12l]
4.3 Comparative Analysis of Grid Current THD

The performance of the two introduced direct MPC with fixed switching frequency strategies (continuous and discontinuous) as well as open-loop CB-PWM and DPWMMIN is investigated and compared in terms of grid current THD in this section. More specifically, the grid current THD is recorded for a wide range of switching frequencies, i.e., from 1650Hz to 4050Hz for continuous schemes and from 1700Hz to 2500Hz for discontinuous schemes.

The grid current THD as a function of average switching frequency is shown in Figure 4.13. The blue lines indicate the performance of CB-PWM in (4.13a) and DPWMMIN in (4.13b) and the dashed red line demonstrates the performance of direct MPC with fixed switching frequency.

As seen in Figure 4.13a, the grid current THD is high at lower switching frequencies and decreases as the switching frequency is increased. Direct MPC with fixed switching frequency achieves similar performance in terms of harmonic distortions as in open-loop CB-PWM with grid current THD being only slightly higher than the theoretical minimum in lower switching frequencies. The discrepancy between the obtained THD in the two schemes at low switching frequencies is partly due to the excited sideband harmonics near the resonance frequency and the presence of low frequency harmonics in the grid current harmonic spectrum. Nevertheless, a lower sampling time for the system can slightly improve the THD for MPC at the expense of more computational time.
As shown in Figure 4.13b when emulating discontinuous modulation, the THD resembles that produced by DPWMMIN even more closely than in continuous scheme. Nonetheless, the controller fails to operate the system at all desired switching frequencies in discontinuous emulation.
5. CONCLUSION

MPC is a method which can address MIMO (non)linear systems without the need for any cascaded loops. Therefore, contrary to conventional linear control methods, it finds a way around the average modeling of the systems and utilizes the instantaneous model instead, thus simplifying analysis and design procedures. Nevertheless, the method usually requires high computational and processing power to solve the optimization problem in short sampling intervals.

In this thesis, the direct MPC algorithm for ac drives proposed in [14]-[15], was adopted for controlling a three-phase two-level grid-connected converter with an $LCL$ filter. Despite the elimination of the modulation stage, it was shown that the controller is able to operate the converter at a fixed switching frequency and produce a discrete harmonic spectrum. This was done by forcing each of the three phase legs to switch once per sampling interval. Additionally, the direct MPC with fixed switching frequency scheme was extended to emulate the $120^\circ$ discontinuous CB-PWM (DPWMMIN).

The harmonic content of the grid current produced by continuous and discontinuous MPC was compared to that produced by open-loop CB-PWM and DPWMMIN, respectively. Moreover, it was shown that for switching frequency $f_{sw} = 2850$Hz in continuous MPC and $f_{sw} = 1900$Hz in discontinuous MPC, the produced grid current harmonic spectra comply with the harmonic limits of the IEEE519 standard. With the preselected $LCL$ filter parameters, the controller was able to successfully operate the converter with the switching frequency to resonance frequency ratio $f_{sw}/f_{res}$ of 2.37 in continuous MPC and 1.58 in discontinuous MPC emulating DPWMMIN.

Under nominal grid conditions, the proposed MPC methods (continuous and discontinuous schemes) achieve similar performance to open-loop CB-PWM and DPWMMIN, in terms of grid current THD. During changes in active and reactive power references, the controller showed fast response and managed to successfully track all the controlled variables. It can be expected that MPC would achieve faster responses during transients. Moreover, with conventional linear control techniques, it
is required that the converter be operated at higher switching frequencies in order not to excite the harmonics around the resonance frequency of the filter. In addition, damping may also be necessary \cite{31}.

5.1 Future Research

In this thesis, the weighting matrix of the objective function, i.e., $\bar{Q}$, held distinct sets of entries during transients. More specifically, different entries for $\bar{Q}$ were chosen for the two operating points and during the transients. Moreover, the weighting matrices were applied for a specific number of sampling intervals selected based on trial and error. This was done to reduce the settling time and avoid any potential overshoots in powers. A more sophisticated way to address this issue which would avoid multiple trial and error attempts could be investigated. E.g., in \cite{32}, input constraints are introduced to address the issue.

The grid-connected converter can also be operated by a three-level neutral point clamped (NPC) inverter, specifically in medium voltage applications \cite{33}. This, leads to lower average switching frequencies and THD. Thus, the performance of the proposed algorithm can be investigated with this type of converter.

The direct MPC algorithm proposed was able to emulate the $120^\circ$ discontinuous modulation scheme (DPWMMIN). In this scheme the switching losses are not distributed equally between the phase legs of the converter. Therefore, investigation of emulating $60^\circ$ and $30^\circ$ discontinuous modulation schemes can be considered.

The performance of the proposed direct MPC with fixed switching frequency algorithm should also be investigated under unbalanced grid conditions.

Finally, the implementation of the direct MPC with fixed switching frequency algorithm on a field programmable gate array (FPGA) is necessary to verify the simulation results.
BIBLIOGRAPHY


APPENDIX A. ORTHOGONAL REFERENCE FRAMES

To simplify the analysis of three-phase systems the variables in the \( abc \)-plane are mapped into a two-dimensional vector in the \( \alpha \beta \)-plane, which is an orthogonal, stationary coordinate system, with its \( \alpha \) axis aligned with the \( a \)-axis.

![Diagram](image)

**Figure 1** \( abc, \alpha \beta \) and \( dq \) coordinates

For a balanced symmetrical three-phase system, the common-mode (zero) component is zero and therefore the transformation can be done via the following matrix, known as the reduced Clarke transformation

\[
\widetilde{K} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2}
\end{bmatrix}
\]

The operation \( \xi_{\alpha \beta} = \widetilde{K} \xi_{abc} \) maps any variable in the \( abc \)-plane to the stationary orthogonal reference frame.

The coefficient \( \frac{2}{3} \) is used to make the transformation amplitude invariant, meaning that the amplitude of the signals in the \( \alpha \beta \)-plane is equal to those in the \( abc \)-plane.

To transform quantities from the \( \alpha \beta \)-plane to the \( abc \)-plane, the inverse Clarke
transformation can be used. The (reduced) inverse Clarke transformation matrix is
given by

$$\tilde{K}^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}.$$  

To map any variable in the $\alpha\beta$-plane to the $abc$-plane, the operation $\xi_{abc} = \tilde{K}^{-1} \xi_{\alpha\beta}$
is used.

An approach to produce dc signals out of ac variables is to rotate the reference
frame. Unlike the stationary reference frame $\alpha\beta$, the rotating reference frame $dq$
rotates counterclockwise with angular speed $\omega_{fr}$.

To transform quantities in $\alpha\beta$-plane to $dq$ quantities, the rotation matrix

$$R(\varphi) = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix},$$

is used. $\varphi$ is the angle between the $d$-axis and the $a$-axis of the $abc$-plane.

To perform the reverse operation, i.e., to turn quantities in the $dq$ frame into quant-
ties in the $\alpha\beta$-plane, the vectors are rotated clockwise by applying the following
matrix

$$R^{-1}(\varphi) = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix}.$$  

The operations $\xi_{dq} = R(\varphi)\xi_{\alpha\beta}$ and $\xi_{\alpha\beta} = R^{-1}(\varphi)\xi_{dq}$ transform any variable from
the $\alpha\beta$-plane to the $dq$-plane and vice versa, respectively.

To directly transform three-phase signals from the $abc$- to the $dq$-plane, Park trans-
formation is used. The Park transformation also results from multiplication of the
$R(\varphi)$ and $\tilde{K}$ matrices introduced above. The (reduced) Park and inverse Park
transformations are respectively given below

$$\tilde{K}(\varphi) = \frac{2}{3} \begin{bmatrix} \cos(\varphi) & \cos(\varphi - \frac{2\pi}{3}) & \cos(\varphi + \frac{2\pi}{3}) \\ -\sin(\varphi) & \sin(\varphi - \frac{2\pi}{3}) & \sin(\varphi + \frac{2\pi}{3}) \end{bmatrix}.$$
\[
\mathbf{K}^{-1}(\varphi) = \begin{bmatrix}
\cos(\varphi) & -\sin(\varphi) \\
\cos(\varphi - \frac{2\pi}{3}) & -\sin(\varphi - \frac{2\pi}{3}) \\
\cos(\varphi + \frac{2\pi}{3}) & -\sin(\varphi + \frac{2\pi}{3})
\end{bmatrix}.
\]

To map any variable directly from the abc reference frame to the dq-plane, the operation \(\xi_{dq} = \mathbf{K}(\varphi)\xi_{abc}\) is used. On the contrary, the operation \(\xi_{abc} = \mathbf{K}^{-1}(\varphi)\xi_{dq}\) is used to transform any variable from the dq-plane to the abc-plane.
APPENDIX B. PER UNIT SYSTEM

To simplify the calculations and normalize the system under study, a per unit system is obtained which expresses the system quantities with respect to their reference (base) values. The per unit value of any quantity is defined as the ratio of the actual value in any unit and the base value in the same unit.

The base values are assumed based on the nominal values of the converter and the grid. The base voltage $V_B$ is the peak value of the nominal phase voltage of the grid, the base current $I_B$ is the peak value of the rated grid current and the base angular frequency is $2\pi f_g$, where $f_g$ is the nominal grid frequency.

The rest of the values are derived based on the three introduced base values and given in Table 1.

<table>
<thead>
<tr>
<th>Base value of</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>$V_B$</td>
<td>$\sqrt{\frac{2}{3}}V_R$</td>
</tr>
<tr>
<td>Current</td>
<td>$I_B$</td>
<td>$\sqrt{2}I_R$</td>
</tr>
<tr>
<td>Angular frequency</td>
<td>$\omega_B$</td>
<td>$\omega_{g,R}$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f_B$</td>
<td>$\omega_B / 2\pi$</td>
</tr>
<tr>
<td>Impedance</td>
<td>$Z_B$</td>
<td>$V_B / I_B$</td>
</tr>
<tr>
<td>Reactance</td>
<td>$X_{L,B}$</td>
<td>$Z_B / \omega_B$</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C_{L,B}$</td>
<td>$1 / \omega_B Z_B$</td>
</tr>
<tr>
<td>Apparent Power</td>
<td>$S_B$</td>
<td>$1.5V_B I_B$</td>
</tr>
</tbody>
</table>

In Table 1, the rated value of the line-to-line grid voltage is denoted by $V_R$. $I_R$ represents the rated grid current and $\omega_{g,R}$ is the rated angular frequency of the grid.